Slope Stability

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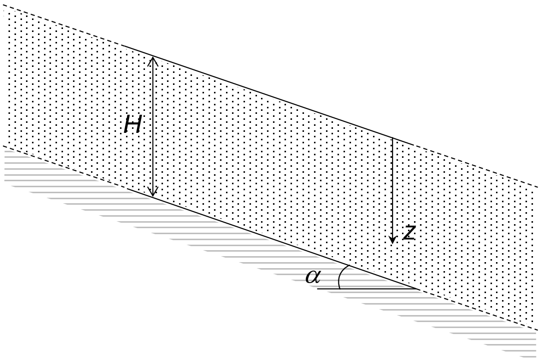
Exercise 2 - Solution

INFINITE SLOPE ANALYSIS

Part 1

Assess the stability of the slope in the figure below considering the different proposed conditions. The following data are given in the corresponding table: saturated unit weight of the soil (γ_{sat}), saturated water content (w_{sat}), geometry of the slope (α and H) and shear strength parameters (φ' and c').



γ_{sat} (kN/m³)	w_{sat} (-)	α (°)	H (m)	φ' (°)	c' (kPa)
21.0	0.235	35.0	3.0	25.0	10.0

1. Compute F considering the following conditions:

   a. dry soil deposit;

   Computation of the dry unit weight (γ_d) of the considered deposit based on the water content and the saturated unit weight.

   The following definitions are valid:

$$\begin{cases} \gamma_{sat} = \dfrac{W_s + W_w}{V_{tot}} \\ \gamma_d = \dfrac{W_s}{V_{tot}} \\ w = \dfrac{W_w}{W_s} \end{cases}$$

   It follows that:

$$\gamma_{sat} = \gamma_d + \frac{W_w}{V_{tot}} = \gamma_d + \frac{wW_s}{V_{tot}} = \gamma_d + w\gamma_d = \gamma_d(1+w)$$

$$=> \quad \gamma_d = \frac{\gamma_{sat}}{1+w}$$

$\gamma_d = \gamma_{sat} / (1+w) = 17$ kN/m^3

The safety factor, F, of a slice can be determined as:

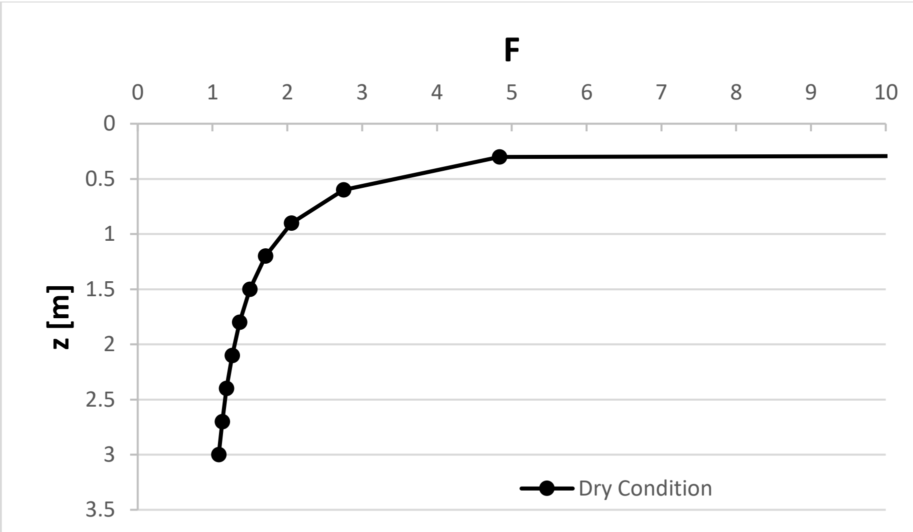
$$F = \frac{T_f}{T} = \frac{c'L}{W'sin\alpha} + \frac{W'cos\alpha\, tan\phi'}{W'sin\alpha} = \frac{c'}{\gamma_d\, z\, sin\alpha\, cos\alpha} + \frac{tan\,\phi'}{tan\,\alpha}$$

Considering that

$$L = \frac{\Delta x}{cos\alpha} \text{ and } W' = z\Delta x\gamma_d$$

Being Δx the length of the considered slice, z the depth of a possible slip surface.

The safety factor F needs to be evaluated for all the possible depths (z) of the slip surface. The depth at which the safety factor is equal 1 is critical. Plotting the values of safety factor corresponding to the different depths, it is possible highlighting that the minimum value is achieved for z=H.



Substituting the available parameters, at z=H we get **F = 1.08**.

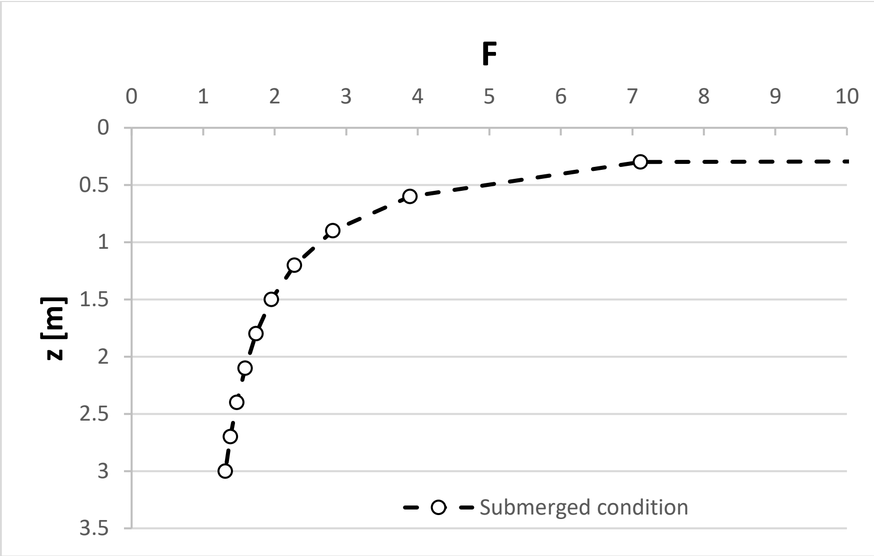
b. submerged soil with a hydrostatic pore water pressure distribution;

The condition of submerged slope with a hydrostatic pore water pressure distribution allows considering that *(i)* the soil is saturated and *(ii)* there is no water flow through the analysis domain. In this case we can refer to the buoyant unit weight γ'.

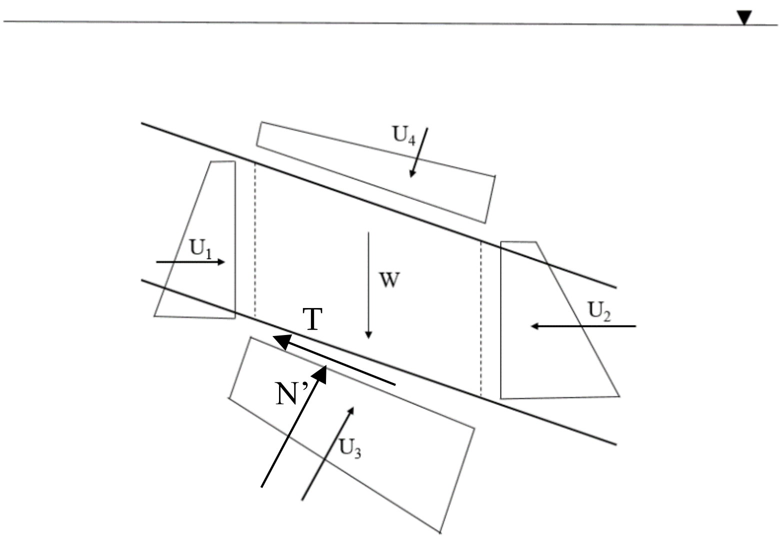
The safety factor, F, can be determined as:

$$F = \frac{T_f}{T} = \frac{c'L}{W'\sin\alpha} + \frac{W'\cos\alpha\tan\phi'}{W'\sin\alpha} = \frac{c'}{\gamma' z\sin\alpha\cos\alpha} + \frac{\tan\phi'}{\tan\alpha}$$

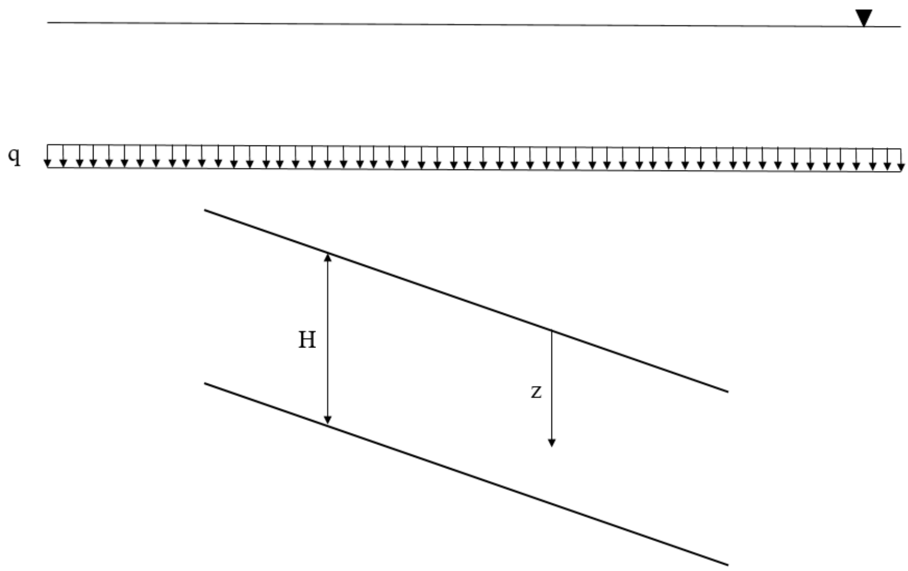
As for the case 1a we can plot F versus the depth. Also in this case F>1 for all the depths investigated; for z=H is: ***F* = 1.30.**



Note that we could have achieved the same value of F considering the equilibrium of the weight of the saturated material W and the resultants of the water pressure distributions.



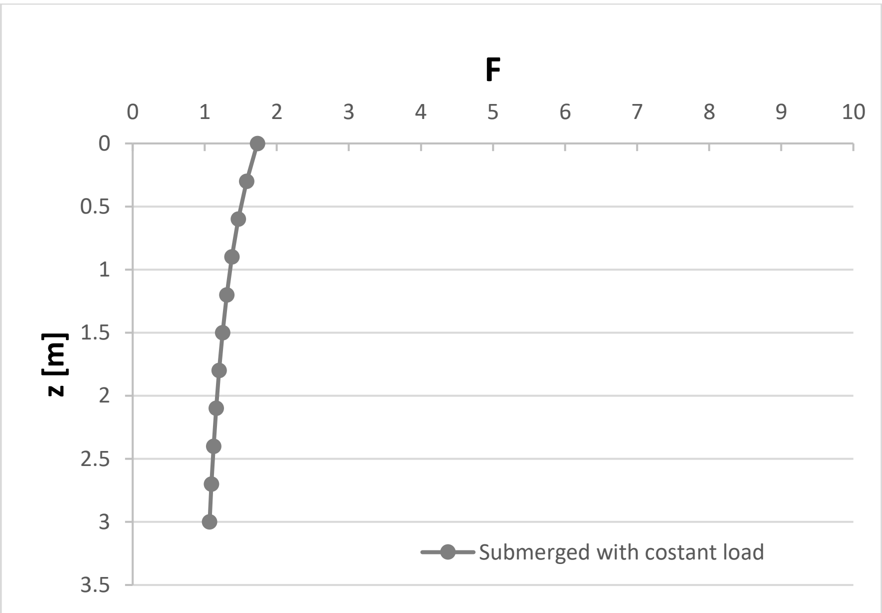
a. as case 1.b, with the additional application of a constant external vertical stress q = 20 kPa acting on the extrados of the slope.



The constant external vertical stress acting on the extrados of the slope can be included in the analysis considering the contribution of an equivalent vertical force per unit of width, *P=q L cosα* acting on the considered slice of soil:

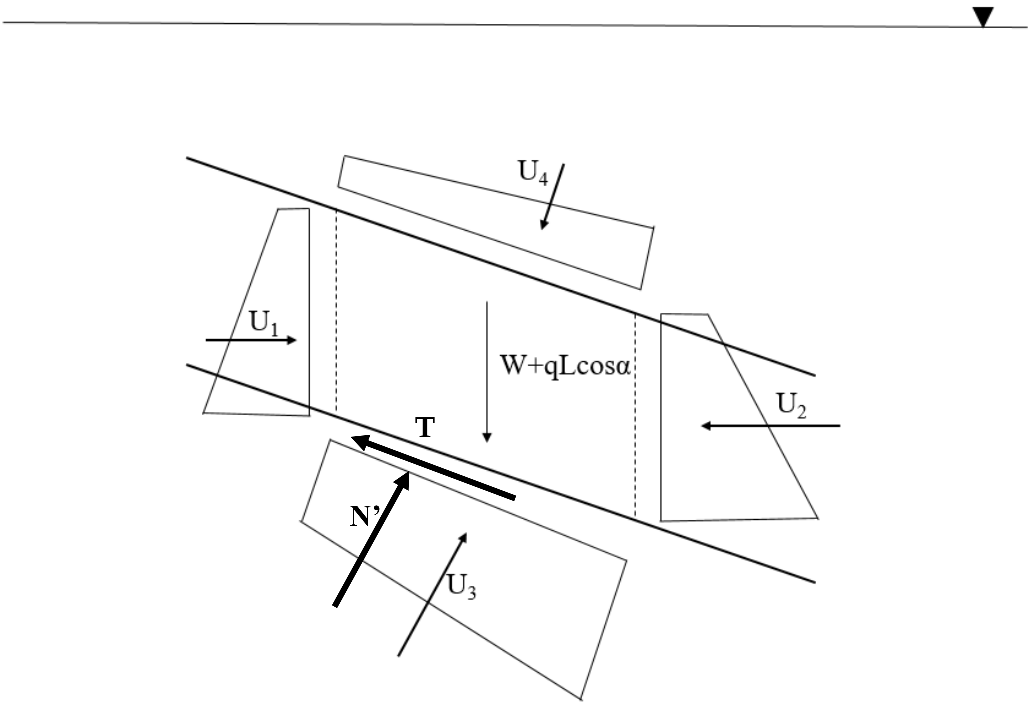
Hence, the factor of safety results to be:

$$F = \frac{T_f}{T} = \frac{c'L}{(W' + qLcos\alpha)sin\alpha} + \frac{(W' + qLcos\alpha)cos\alpha\ tan\phi'}{(W' + qLcos\alpha)sin\alpha} = \frac{c'}{(\gamma' z + q)sin\alpha\ cos\alpha} + \frac{tan\phi'}{tan\alpha}$$



Substituting the available data, at z=H we get ***F* = 1.06**.

It is important to remark that, in this case too, we could achieve the same value of F referring to the saturated weight W but considering in the equilibrium also the resultants of the water pressure distributions and the resultant of the vertical load.

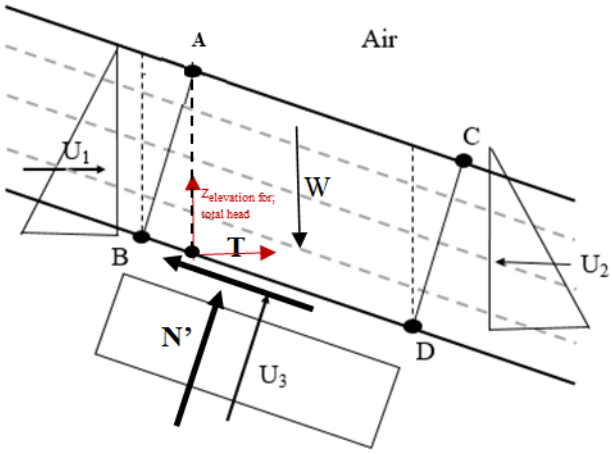


2. For a flow parallel to the slope, compute F at $z = H$ using two alternative approaches to account for the water flow: (i) total weight of the soil and pore water pressure distributions along the sides of the soil slice, (ii) buoyant weight of the soil and seepage force.

   (i) For seepage parallel to slope, considering the total weight of the soil and pore water pressure distributions along the sides of the element, the factor of safety can be computed as:

$$F = \frac{T_f}{T} = \frac{c'}{\gamma_{sat}\, z\, sin\alpha\, cos\alpha} + \frac{\gamma'}{\gamma_{sat}}\frac{tan\,\phi'}{tan\,\alpha}$$

In fact, if the flow is parallel to the slope, we can define the equipotential lines AB and CD ($h=z_e+p/\gamma_w$ = cost). With respect to the given reference system z_e, we obtain:



$$h_A = z_A + \frac{P_A}{\gamma_w} = H \quad => \quad h_B = H$$

A and B belong to the same isopiezic line, $h_B = h_A$; it follows:

$$P_B = \gamma_w (h_B - z_B) = \gamma_w \ (H - H\sin^2\alpha) = \gamma_w \, H \ \cos^2\alpha$$

Similarly, for the equipotential line CD.

It follows that:

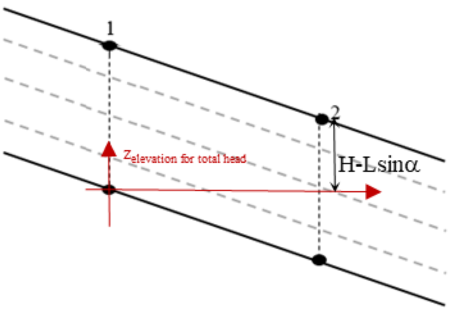
$$\begin{cases} U_1 = U_2 \\ U_3 = \gamma_w \, HL \ \cos^2\alpha \\ W = \gamma_{sat} \, HL\cos\alpha \end{cases}$$

Hence:

$$F = \frac{T_f}{T} = \frac{c'}{\gamma_{sat} \, H \, sin\alpha \, cos\alpha} + \frac{\gamma'}{\gamma_{sat}} \frac{tan \, \phi'}{tan \, \alpha}$$

Substituting the available data, we get ***F* = 0.69**. The equilibrium is not verified; this means that the failure surface for which F=1 is located at a depth $z_{crit} < H$.

(ii) The same result can be obtained considering the buoyant weight of the soil, *W*', and the seepage force, *J*.



$$W' = \gamma' HL\cos\alpha \qquad\qquad J = \gamma_w iV$$

Where i is the falling head (h1 and h2 are the total heads in 1 and 2 respectively):

$$\begin{cases} h_1 = z_1 + \dfrac{P_1}{\gamma_w} = H \\ h_2 = z_2 + \dfrac{P_2}{\gamma_w} = H - L\sin\alpha + 0 = H - L\sin\alpha \\ i = \dfrac{\Delta H}{L} = \dfrac{h_1 - h_2}{L} = \dfrac{L\sin\alpha}{L} = \sin\alpha \end{cases}$$

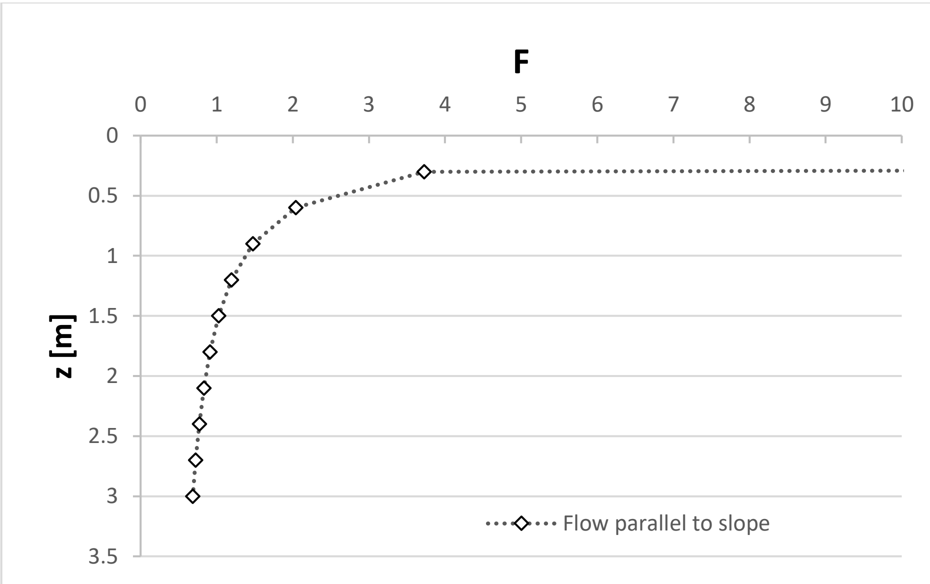
From the equilibrium:

$$\begin{cases} N' = \gamma' HL \cos^2\alpha \\ T = \gamma' HL\cos\alpha\sin\alpha + \gamma_w HL\cos\alpha\sin\alpha \end{cases}$$

$$F = \frac{T_f}{T} = \frac{c'L + \gamma' HL \cos^2\alpha\tan\phi'}{\gamma' H L\, sin\alpha\, cos\alpha + \gamma_w HL\cos\alpha\sin\alpha} = \frac{c'}{\gamma_{sat} H\, sin\alpha\, cos\alpha} + \frac{\gamma'}{\gamma_{sat}}\frac{tan\,\phi'}{tan\,\alpha}$$

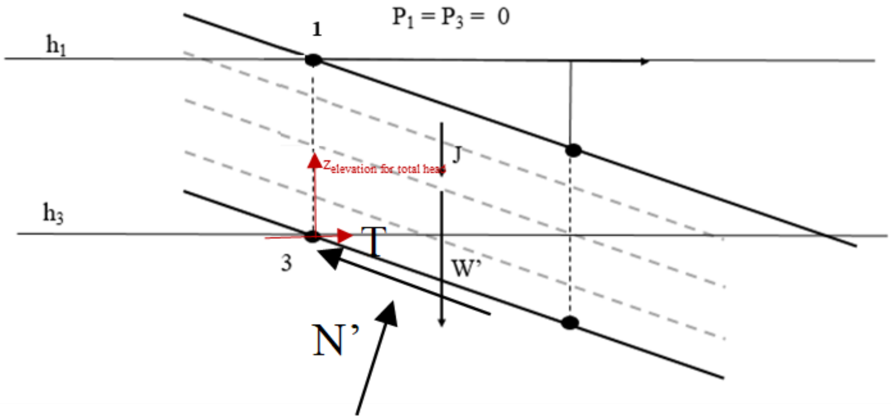
Thus: $F = \dfrac{T_f}{T}$ = **0.69**

At z=H the equilibrium is not verified, this means that the failure surface is located at a depth $z_{crit} < H$.



3. Compute F at $z = H$ in the case of a vertical flow.

It is decided to follow the approach (ii), in this case the seepage force is vertical:



$W' = \gamma' HL \cos\alpha$ $\qquad\qquad J = \gamma_w iV$

$$\begin{cases} h_3 = z_3 + \dfrac{P_3}{\gamma_w} = 0 \\ h_1 = z_1 + \dfrac{P_1}{\gamma_w} = H + 0 = H \\ i = \dfrac{\Delta H}{H} = \dfrac{h_1 - h_3}{H} = \dfrac{H}{H} = 1 \end{cases}$$

where i is the falling head, h_1 and h_3 are the total heads in 1 and 3 respectively.

Thus,

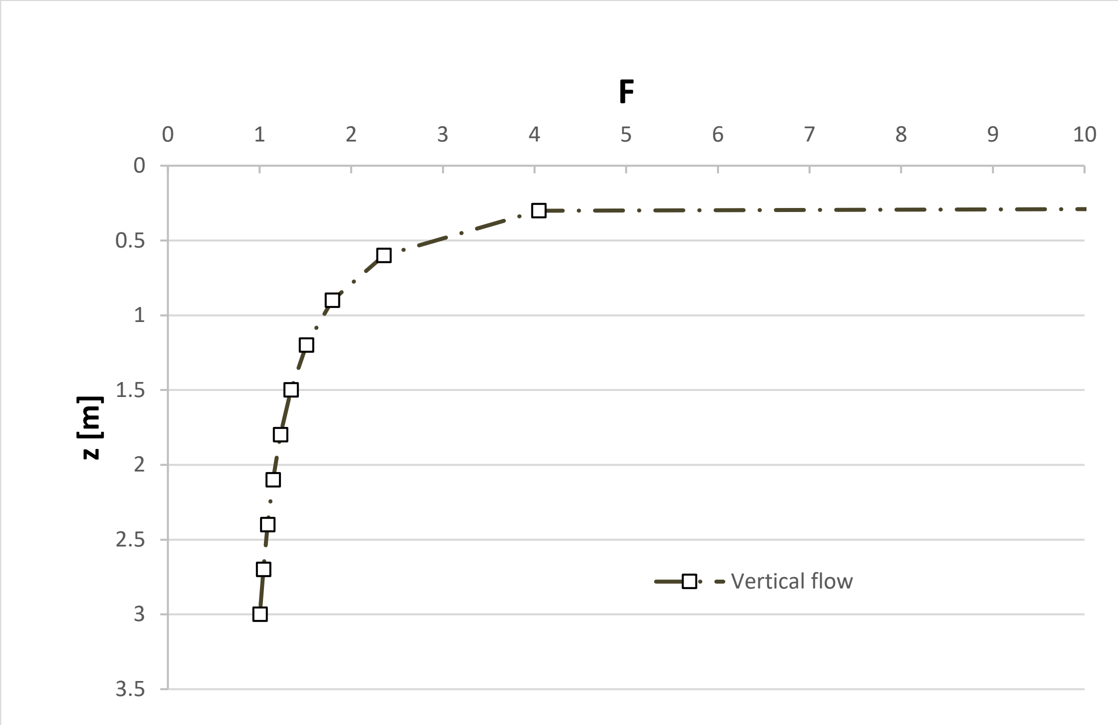
$$\begin{cases} N' = (W' + J)\cos\alpha = \gamma' HL \cos^2\alpha + \gamma_w HL\cos^2\alpha \\ T = (W' + J)\sin\alpha = \gamma' HL\cos\alpha\sin\alpha + \gamma_w HL\cos\alpha\sin\alpha \end{cases}$$

So,

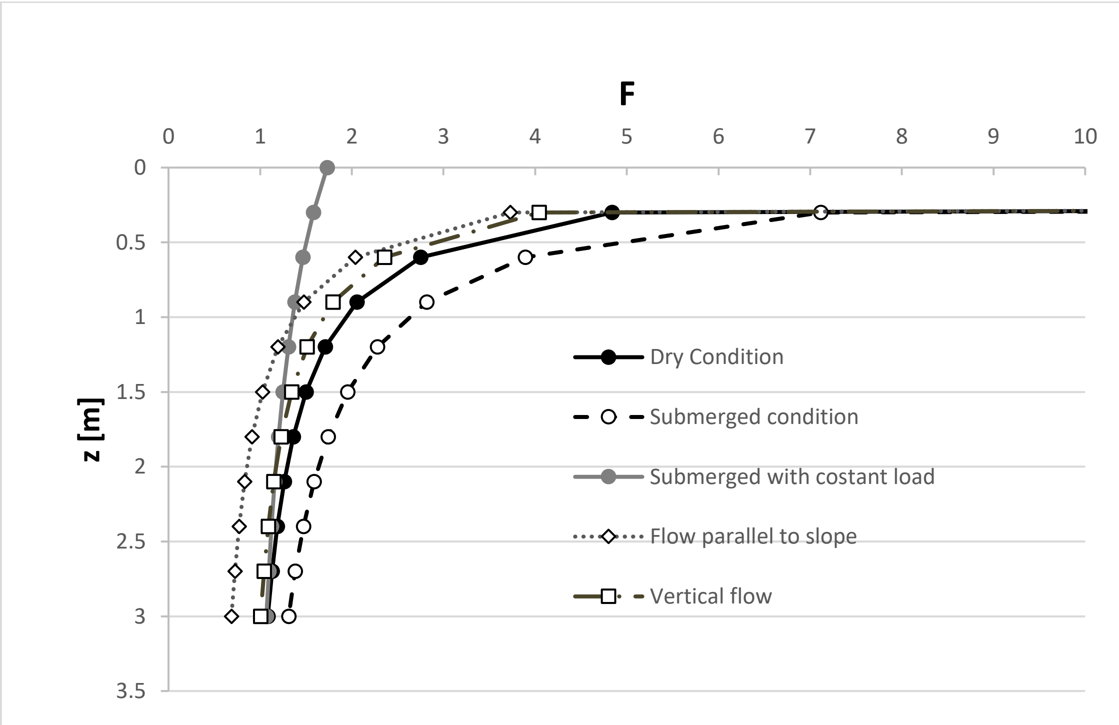
$$F = \frac{T_f}{T} = \frac{c'L + (\gamma' HL \cos^2\alpha + \gamma_w HL\cos^2\alpha)\tan\phi'}{\gamma' H L \, sin\alpha \, cos\alpha + \gamma_w HL\cos\alpha\sin\alpha} = \frac{c'}{\gamma_{sat} H \, sin\alpha \, cos\alpha} + \frac{tan\,\phi'}{tan\,\alpha}$$

For z=H the safety factor is:

$$F = \frac{T_f}{T} = \mathbf{1.00}$$

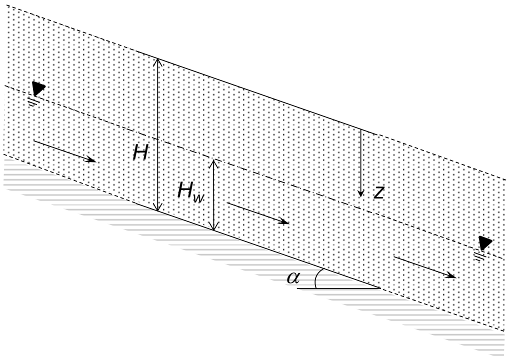


It is plotted the variation of safety factor over depth for all the previous cases:



Part 2

The sand deposit in the figure below is characterized by a water flow parallel to the slope; the water table is below the ground level. The soil above the water table can be considered dry.

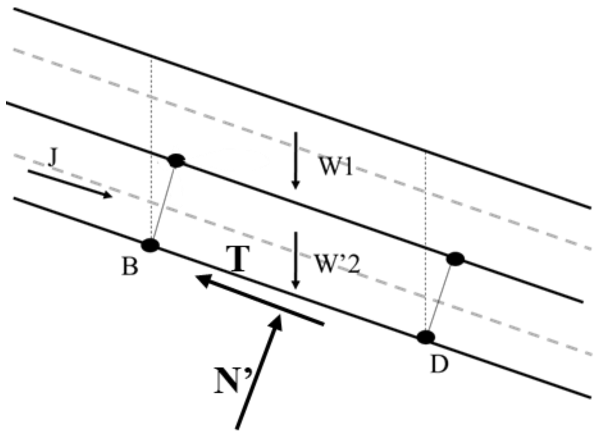


γ_{sat} (kN/m^3)	w_{sat} (-)	α (°)	H (m)	φ' (°)	c' (kPa)
22.0	0.25	25.0	3.0	35.0	0

4. In order to design a possible intervention on the level of the water table, compute the maximum Hw to guarantee F ≥ 1.3.

To solve this case, we adopt the approach (ii).

We compute the weight of the soil submerged W_2' (below the water table) and dry W_1 (above the water table) separately.



$$W_1 = \gamma_d (H - H_w) L \cos\alpha$$

$$W_2' = \gamma' H_w L \cos\alpha$$

The seepage force, J, can be computed as:

$$J = \gamma_w i V = \gamma_w i H_w L \cos\alpha$$

For the case of the flow parallel to the slope, the following relation is valid

$$i = \sin\alpha$$

Therefore, we obtain

$$\begin{cases} N' = \gamma_d (H - H_w) L \cos^2\alpha + \gamma' H_w L \cos^2\alpha \\ T = \gamma_d (H - H_w) L \cos\alpha \sin\alpha + \gamma' H_w L \cos\alpha \sin\alpha + \gamma_w H_w L \cos\alpha \sin\alpha \end{cases}$$

The safety factor can be expressed as:

$$F = \frac{\left\{\left[\gamma_d (H - H_w) + \gamma' H_w\right]\cos\alpha \tan\phi' + \frac{c'}{\cos\alpha}\right\}}{\left\{\left[\gamma_d (H - H_w) + \gamma' H_w\right]\sin\alpha + \sin\alpha H_w \gamma_w\right\}}$$

By imposing the condition that:

$$F = \frac{\left\{\left[\gamma_d (H - H_w) + \gamma' H_w\right]\cos\alpha \tan\phi' + \frac{c'}{\cos\alpha}\right\}}{\left\{\left[\gamma_d (H - H_w) + \gamma' H_w\right]\sin\alpha + \sin\alpha H_w \gamma_w\right\}} \geq 1.3$$

We obtain that to have a safety factor $F \geq 1.3$ it is necessary to have:

$H_w \leq$ **0.75 m**